Towards a framework for making effective computational choices: A ‘very big idea’ of mathematics

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It is important for students to make informed decisions about computation. This article highlights this importance and develops a framework which may assist teachers to help students to make effective computational choices.

Setting the scene

In the same way that the purpose of counting is to quantify, perhaps it could be said that the purpose of much of the mathematics learned in school is to allow us to calculate answers to problems. This may sound rather utilitarian but the intent is not to underplay more affective aspects of mathematics. Rather it is meant to underline the importance of knowing how to make sound choices about calculations and that computational choice is indeed, ‘a very big idea’ of mathematics. ‘Big idea’ thinking in mathematics is not new and Charles (2005) used the term to highlight the importance of seeing the many links and connections within and between big ideas. He described a big idea as “a statement of an idea that is central to the learning of mathematics, one that links numerous mathematical understandings into a coherent whole” (Charles, 2005, p. 10). He listed 21 big ideas, many of which could generally be described as being content-based but some of which were broader in nature such as equivalence, pattern, and comparison. The Australian Association of Mathematics Teachers (2009) noted the importance of big ideas and separated content ideas such as shape and space, quantity, and uncertainty from big ideas such as those just mentioned but also including ideas like dimension, transformation, and algorithm. This paper puts forward the view that computational choice is a big idea that requires careful development.

Background

In an earlier article in this publication, Swan (2004, p. 27) noted that various mathematics curriculum documents of the time highlighted “the need for students to choose from a repertoire of computational tools and strategies” but lamented the fact that “little direction is given as to how children make the choice as to which form of computation to use in any given situation”. Twelve years on, the situation has not changed. A scan of the *Australian Curriculum: Mathematics* Version 8.1 (ACARA, 2015) reveals that the word ‘choice’ is used just four times in the K–10 curriculum. Indeed, it is not until the Year 7 content descriptions—in the elaborations about ACMNA157—that mention is made of choice in connection with computations.

- Justifying choices about partitioning and regrouping numbers in terms of their usefulness for particular calculations. ACMNA053 Year 3
- Identifying the best methods of presenting data to illustrate the results of investigations and justifying the choice of representations. ACMSP119 Year 5
- Justifying choices of written, mental or calculator strategies for solving specific problems including those involving larger numbers. ACMNA157 Year 7
- Understanding that quantities can be represented by different number types and calculated using various operations, and that choices need to be made about each. ACMNA157 Year 7 (ACARA, 2015).

The *National Statement on Mathematics* (Australian Education Council, 1991) noted that students need to be able to make decisions about operations and how to carry them out before performing the operation and making sense of the answer. Sadly, such details do not appear in the current curriculum document.

The indicator “Justifying choices of written, mental or calculator strategies for solving specific problems including those involving larger numbers” is a good statement but it makes no progress towards indicating the knowledge, skills, attitudes and abilities needed by children to actually make effective choices. In fact, the statement could easily be missed (or dismissed) or misinterpreted by teachers.

Some evidence

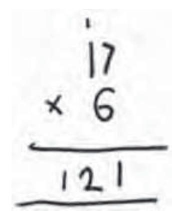
Teachers need to be assisted to interpret the curriculum document, to ‘read between the lines’, and to ask themselves “What do children need to know, understand, and be able to do in order to make effective computational choices, and to justify those choices?” Before attempting to answer that important question, I want to illuminate the situation by presenting some data from a current research project into children’s multiplicative thinking. Even in a relatively small sample, it is apparent that students approached the task in several different ways, not all of which were successful. A group of twenty-two Year 5 students from one school and sixteen Year 6 students from a different school were asked individually to offer a solution to 6 × 17 and to explain how they arrived at their answer. A range of materials including bundling sticks and MABs were available for them to use. Nineteen students (one Year 6 and eighteen Year 5) chose a mental strategy and nineteen (fifteen Year 6 and four Year 5) chose a written strategy.

Of the 19 who worked mentally, 15 promptly obtained a correct answer. When asked to explain how they did it, they did so in terms of standard place value partitioning (6 × 10 and 6 × 7) or they used an alternative method. Four of the students who attempted to use a mental strategy made an error but when probed further, each was able to either self-correct or explain the process in terms of standard place value partitioning. Many were also able to correctly show the example using bundling sticks, that is, they were able to show six groups of seventeen sticks. Some typical examples of explanations follow here.

> Jimmy described his working as “Six times seven is 42 and six times one is six, so I added 42 and 60 to get 102”. [Interviewer: “Where did the 60 come from?”] “Six times one is actually a ten so it’s sixty”. He was also able to use MABs to show what happened in the working out.

> Joey said he ‘mentally split the number’—“I took away the seven and did six times 10 equals 60, then six times seven is 42 and added them to get 102”. He used bundling sticks saying “I would have six groups and there would be 17 in each group”. He made a group of 17 with a bundle of 10 and seven singles and said “You would have five more groups like that”.

There were 19 students who chose a written strategy and ten obtained a correct answer. However, when probed further, none of the nine who made an error was able to self-correct or arrive at an explanation based on partitioning. Also, two of the students who did obtain a correct answer with a written method were unable to explain what they actually did. Some examples of explanations follow.



> Caty did the vertical algorithm (changed order of the numbers) and obtained 121. She explained “Six goes into seven once, write down one and carry one, and add it to the one in the 17. Six times two is 12 and the answer is 121”.

> When given the exercise, Molly asked “Can I write stuff down?” She obtained an incorrect answer. She explained “You make a times table type thing—six times seven is 52, you put the two down and carry the five up there. You have to add that to the other number you’re timesing”. [Interviewer: “What’s the five you carried really worth?”] “It’s five”. When she checked the answer on a calculator, she initially couldn’t see where she went wrong. She said she couldn’t do it in her head and when probed further she said “I’d do seven times six is 42 and then one times six”. [Interviewer: “One times six?”] “Yes”.

There are clear differences between the responses of children who chose to work mentally and the children whom used a written method. All of the 19 students who worked mentally either obtained a correct answer, and/or could explain conceptually what they did, and/or self-corrected. Less than half the students who chose a written method obtained a correct answer and explained their working appropriately.

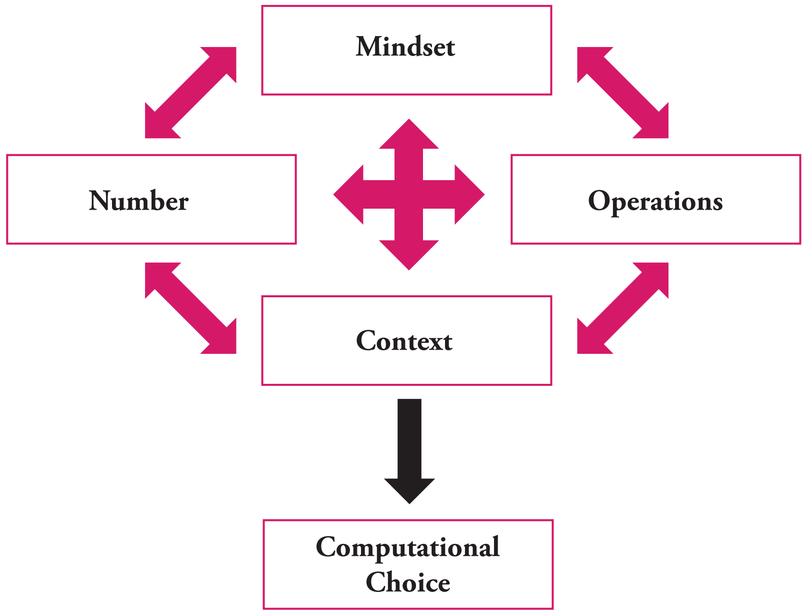


Figure 1. A framework for computational choice mindset.

What conclusions can be drawn from this? Could it be that students using the written algorithm were taught to use it as a 'first resort'? Had they been taught the algorithm without having the underpinning conceptual understanding of place value and partitioning? Is it likely that they had been exposed to a range of mental strategies for calculating one-digit by two-digit multiplication examples? In short, did they make an effective choice about computational strategies?

Seeking a framework for computational choice

In returning to the original question posed earlier, "What do children need to know, understand, and be able to do in order to make effective computational choices, and to justify those choices?", it is useful to consider the *Number Sense Framework* developed by McIntosh, Reys and Reys (1997). They proposed that there were three key elements to number sense —a facility with numbers, a facility with operations, and the ability to apply both to computational settings. These elements could be described as being mathematical in nature in that they are to do with content and concepts, and can be adapted to form the basis for a Computational Choice Framework—that is, number, operations, and context. However, there would seem to be a collection of other factors that influence a child's capacity to make effective computational choices—that is the affective component of mindset. The related notions of confidence and anxiety towards mathematics have been acknowledged and researched (Hurst & Cooke, 2012) and it seems logical to suggest that similar factors may come into play when children are faced with making choices about computations. To some extent, context is affective in nature (like mindset) while the other two components of number and operations are more specifically mathematical. Indeed, the components of context and mindset could easily be applied to other learning areas. The framework is represented in Figure 1 where the four components of mindset, number, operations, and context act upon one another. Context could be described as leading to the action of making the computational choice. Each of the components will be described in turn and, given the scope of this paper, not all aspects of each component are discussed in detail.

The assertion made here is that 'mindset' or 'disposition' can be a powerful influence on a student's capacity to make effective choices, notwithstanding his or her understanding of number and operations.

Sparrow and Swan (2006) described a similar idea which they called 'metacomputation', this being derived from metacognition, or 'thinking about one's thinking'. It involves teachers explicitly modelling thinking when working on a calculation. Sparrow and Swan (2006, p. 144) noted that "novice calculators are not given access to how people approach calculating situations" and that explicit modelling of thinking "is different from teaching a specific procedure for a calculation". Clearly there are specific aspects of thinking based on number such as "What do I know about this number?", "How are these numbers connected or related?" and "What will happen when I multiply this number

that ends in a five?” However, it is suggested that the notion of mindset is broader and deeper than meta-computation and that there are many more generic habits of mind and attitudes that children need to be encouraged to develop. Some typical ideas are listed below.

- I should have a rough idea of what sort of answer I am expecting to get *before* I calculate.
- When doing a calculation with whatever method I choose, I should always estimate to see if I’m on the right track and at the end to see if my answer makes sense.
- There are ways of checking to see if an answer is right or not.
- There may be a variety of methods other than the one I choose to do a particular calculation.
- My own method of working out something might be better than others but sometimes other methods might be better than mine.
- The quickest way to get an answer may not be the best way.
- The longest way to get an answer may not be the best way.
- If the method I’ve chosen doesn’t seem to be working, it is all right to start using a different method.
- My answer could be right or wrong. I cannot assume that it is right.
- When calculating with numbers, I should only do what makes sense. If I don’t understand *why* I am doing something, I shouldn’t do it.
- Working mentally doesn’t mean working entirely in the head. I should record and write whatever I need to help me understand what I am doing.
- It is all right to make a mistake—I can learn from it.
- I cannot assume that an answer produced by a calculator is correct. A calculator will do what I tell it to do. If I enter something incorrectly, the calculator will not correct it for me.
- If I correctly enter details and commands into a calculator, it will produce the correct answer (unless it has an electronic fault or a hardware fault).
- I should check at each stage of making a calculator entry that I have entered what I intended to enter.
- I can use/recall previous things I know and have done in order to help me.
- There are certain things I know about maths that can help me work out other things.
- I can share ideas with others to help me (and them) learn better.

Number

There are many important ideas about number that inform students’ capacity to make effective choices, such as a flexible understanding of place value and the base ten number system, including the ideas of partitioning and regrouping. These underpin the understanding of written and mental strategies, as do the recall of, and facility with, basic and extended number facts, the ability to recognise and make connections between them, and to derive unknown number facts. These can be considered as the mathematical ‘nuts and bolts’ of computation. The number component is shown in Figure 2.

Place value, base ten system	Landmark and benchmark numbers	Basic number facts
Trading, regrouping	**How numbers are structured, built, made-up**	Extended number facts
Flexible partitioning, chunking	Relative magnitude of numbers and ordering	Patterns in numbers
Regrouping, trading up and down	Fractions and decimals	Known facts > derived facts Double and half relationships

Figure 2. Number component of a computational choice framework

Operations

The number component contains many essential ideas which underpin computational choice, as does the operations component. The main ideas here centre on the conceptual understanding of the additive and multiplicative situations. The additive situation is about the part-part-whole relationship just as the multiplicative situation is about the idea of a number of equal groups and the factor-multiple relationship. In order to make effective computational choices students need explicit exposure to a range of additive and multiplicative problem types. For additive problems, this entails working with ‘result unknown’, ‘change unknown’, and ‘start unknown’ problems and for multiplicative problems, working with ‘equal group problems’, ‘comparison problems’, ‘combination problems’, and ‘array problems’ (Van De Walle, Karp, & Bay-Williams, 2013).

It is important for them to understand the similarities that exist. For example, if we know the two parts of an additive situation, we add to find the total and if we know one part and the total, we subtract. Similarly, for multiplicative equal group problems, if we know the group size and number of groups, we multiply to find the total, but if we know the total and one of the group factors, we divide to find the other. The operations component is shown in Figure 3.

The multiplicative situation	Effect of operating on numbers. Will answer be larger/ smaller?	The additive situation
Group size and number of groups	**What happens when we operate with numbers**	Part-part-whole relationship
Properties– commutative, distributive	Rounding and estimation	Properties– commutative, associative
Factors, multiples		Links between operations

Figure 3. Operations component of a computational choice framework.

Context

The component of context is about the particular way in which a computation is situated or embedded. It is important that students are provided with ample opportunities to experience a wide range of contexts and to be shown explicitly how to use aspects of the context to help them make effective computational choices. Some of the considerations of which students need to explicitly be made aware are as follows.

Required degree of accuracy

- Can I estimate?
- Is a 'ball park' range good enough?
- Is it all right to be 'fairly close' to the answer?
- Do I have to be very accurate or exact?
- Does it have to be 'right' or 'wrong'? If I'm hypothesising, does it matter?

Method

- Should I use a provided method, my method, or someone else's invented method?
- Should I do it mentally?
- Is it all right to work in the head or with the head?
- Should I use informal jottings while working with the head?
- Is it best to use a written procedure with pen and paper?
- Will a calculator help me? How?
- Is it best to use a combination of all of the above?

The mathematics

- Which operation is it? Add/subtract/multiply /divide?
- Is it a combination of several? How do I know?
- Have I tried to write a number sentence to match the problem?
- Is there anything that I recognise from a similar situation/problem?

Other considerations

- Does it involve fractions? Should I change them to decimals or percentages?
- What could I do to the numbers? Regroup? Change the order?
- How do I know which information in the problem is relevant, necessary, superfluous?

Units/instruments and resources

- Which units/instruments do I use? Why? How do I know?
- Do I have to convert units or not? Why? Why not? How?
- What are the most appropriate units/ instruments to use? How do I know?
- Are there any manipulatives, tools and resources that I can use to help me?
- Can I/should I do a drawing?

Other factors that might have an impact on students' capacity to make effective computational choices include language aspects of the context in which a problem might be embedded. Again, explicit teaching around such issues is important, particularly for children who experience language difficulties or who come from an ESL background. These factors are shown in Figure 4.

Everyday language as used in maths		Social and peer group language
Multiple meanings (e.g., volume)	**What happens when we operate with numbers**	Language of textbook problems
	Mathematical language and terminology	

Figure 4. Other factors in the context component of a computational choice framework.

Conclusions

Swan and Sparrow (2006) made the point that explicit teaching and modelling of thinking strategies is vastly different to the teaching of procedures, which, without the underpinning conceptual understanding, is at best unhelpful. The evidence presented earlier suggests that perhaps some of the students in the sample may well have been taught a written algorithm for multiplication without such understanding whereas others are likely to have been exposed to explicit teaching of mental strategies, strengthened by clear and effective teaching of elements of the number component of the model, namely place value and flexible partitioning. It is also possible that students who are well equipped to make effective computational choices have been allowed to develop and use their own 'student-invented strategies'. Van De Walle et al. (2013, p. 218–219) outline the benefits of such strategies as being "number oriented rather than digit oriented" and "are a range of flexible options rather than being 'one right way'". They also claim that students make sense of the mathematics when they develop their own methods, they make fewer errors, they are often faster, and there is less re-teaching required (Van De Walle et al., 2013).

Computational choice is certainly a big idea of mathematics which is identified in the *Australian Curriculum: Mathematics* as being important. As Swan (2004) pointed out some years ago, it is not made sufficiently clear to teachers just *how* important it really is, and *how* they should go about developing it in their students. This paper has attempted to illuminate the situation by suggesting a Computational Choice Framework that includes affective elements as well as more clearly mathematical elements. It is the author's intention to use it as the conceptual framework for a research project to investigate what factors influence children's computational choices.

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